

MPC for Robot Manipulators With Integral Sliding Modes Generation

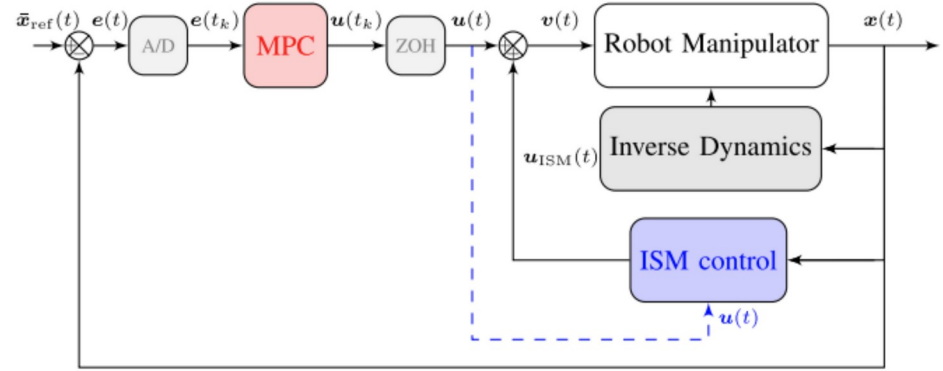
Incremona, Ferrara, Magni; IEEE/ASME Transactions on Mechatronics



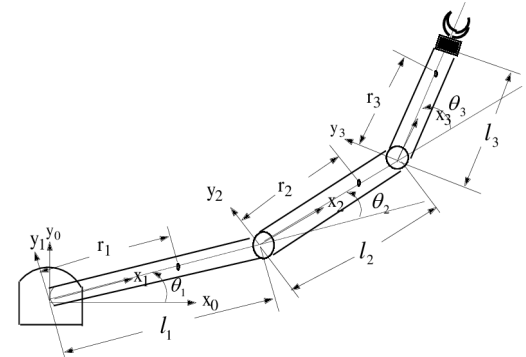
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Problem Statement

- MPC struggles with robot systems due to:
 - Nonlinear MIMO dynamics
 - Modelling uncertainties
 - High computational cost
- Proposed Solution:
 - Hierarchical control: Inverse Dynamics + ISM + MPC
 - ISM handles uncertainties first
 - Leads to a simplified MPC optimization
- Key Novelty:
 - ISM enforces sliding modes from initial time, creating uncertainty-free dynamics
 - Multi-rate control (fast ISM, slower MPC)
 - Significantly reduced computational complexity



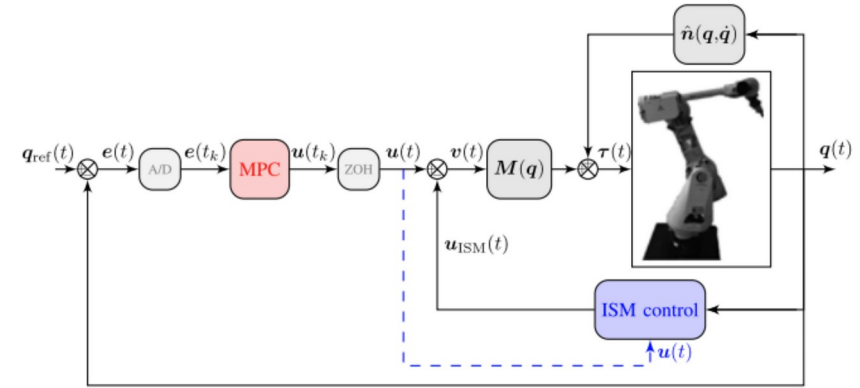
$$M(q)\ddot{q} + h(\dot{q}, q) + g(q) = F(t)$$



Controller Overview

Three Layer Control Architecture

- 1) Inner Loop: Inverse Dynamics Controller
 - a) Linearizes nonlinear MIMO system
 - b) Transforms to n decoupled double integrators
- 2) Middle Loop: Integral Sliding Mode (ISM)
 - a) Rejects matched uncertainties
 - b) Fast sampling rate (high frequency)
- 3) Outer Loop: MPC
 - a) Optimal trajectory tracking
 - b) Handles constraints
- 4) Combine Control Law:
 - a) $v = u_{\text{MPC}} + u_{\text{ISM}}$



The full coupled robot model:

$$\underbrace{M(q)}_{\text{inertia matrix}} \ddot{q} + \underbrace{n(q, \dot{q})}_{\text{Coriolis/centripetal gravity + friction}} = \tau$$

$$e = q_{\text{ref}} - q$$

$$\begin{aligned}
I_2 &= I_{222} + m_2 * (r_{12}^2 + r_{13}^2) + (m_3 + m_4 + m_5 + m_6) * a_2^2 \text{ ;} \\
I_3 &= -I_{222} + I_{223} + (m_3 + m_4 + m_5 + m_6) * a_3^2 \text{ ;} \\
&\quad m_3 * r_{23}^2 - m_2 * r_{12}^2 \text{ ;} \\
I_4 &= m_2 * r_{23} * (d_2 + r_{12}) + m_3 * a_2 * r_{13} \\
&\quad + (m_3 + m_4 + m_5 + m_6) * (d_2 + d_3) \text{ ;} \\
I_5 &= -m_2 * r_{13} * r_{23} + (m_4 + m_5 + m_6) * a_3 + d_4 + m_4 * a_2 * r_{14} \text{ ;} \\
I_6 &= I_{222} + m_2 * r_{15}^2 + m_4 * a_2^2 + m_4 * (d_4 + r_{14})^2 + I_{221} \\
&\quad + m_5 * a_3^2 + m_5 * d_4^2 + I_{233} + m_6 * a_5^2 + m_6 * d_5^2 \\
&\quad + m_6 * r_{16}^2 + I_{226} \text{ ;} \\
I_7 &= m_2 * r_{12}^2 + I_{222} - I_{233} + m_4 * r_{14}^2 + 2 * m_4 * d_4 * r_{14} \\
&\quad + (m_4 + m_5 + m_6) * (d_2^2 - a_2^2) + I_{231} - I_{224} + I_{225} \\
&\quad - I_{235} + m_6 * r_{16}^2 - I_{216} + I_{236} \text{ ;} \\
I_8 &= -m_4 * (d_2 + d_3) * (d_4 + r_{14}) - (m_3 + m_6) * (d_2 + d_3) * d_4 \\
&\quad m_3 * r_{13} * r_{23} + r_{23} * m_3 * (d_2 + d_3) * r_{13} \text{ ;} \\
I_9 &= m_2 * r_{12} * (d_2 + r_{12}) \text{ ;} \\
I_{10} &= 2 * m_4 * a_2 * r_{14} + 2 * (m_4 + m_5 + m_6) * a_3 * d_4 \text{ ;} \\
I_{11} &= -2 * (m_2 * r_{23} * r_{12} \text{ ;} \\
I_{12} &= (m_4 + m_5 + m_6) * d_2 * a_3 \text{ ;} \\
I_{13} &= (m_4 + m_5 + m_6) * a_3 * (d_2 + d_3) \text{ ;} \\
I_{14} &= I_{224} + I_{225} + I_{116} \text{ ;} \\
I_{15} &= m_4 * d_4 * r_{16} \text{ ;} \\
I_{16} &= m_6 * a_5 * r_{16} \text{ ;} \\
I_{17} &= I_{223} + I_{236} + m_6 * r_{16}^2 \text{ ;} \\
I_{18} &= m_6 * (d_2 + d_3) * r_{16} \text{ ;} \\
I_{19} &= I_{224} - I_{224} + I_{115} - I_{233} + m_6 * r_{16}^2 + I_{226} - I_{116} \text{ ;} \\
I_{20} &= I_{225} - I_{225} - m_6 * r_{16}^2 + I_{116} - I_{226} \text{ ;} \\
I_{21} &= I_{234} - I_{234} + I_{225} - I_{115} \text{ ;} \\
I_{22} &= m_6 * a_5 * r_{16} \text{ ;} \\
I_{23} &= I_{116} \text{ ;}
\end{aligned}$$

Part II. Gravitational Constants

$$\begin{aligned} g_1 &= -g * \{(m_2 + m_4 + m_5 + m_6) * a_2 + m_2 * r_{22}\} \frac{1}{2} \\ g_2 &= g * \{m_2 * r_{23} - (m_4 + m_5 + m_6) * d_4 - m_4 * r_{14}\} \frac{1}{2} \\ g_3 &= g * m_2 * r_{32} \frac{1}{2} \\ g_4 &= -g * \{m_4 + m_5 + m_6\} * a_3 \frac{1}{2} \\ g_5 &= -g * m_6 * r_{46} \frac{1}{2} \end{aligned}$$

Table A3. Computed Values for the Constants Appearing in the Equations of Forces of Motion.

(Inertial constants have units of kilogram meters-squared)

$I_1 = 1.43 \pm 0.05$	$I_2 = 1.75 \pm 0.07$
$I_3 = 1.38 \pm 0.05$	$I_5 = 6.90 \times 10^{-1} \pm 0.20 \times 10^{-1}$
$I_4 = 3.72 \times 10^{-1} \pm 0.31 \times 10^{-1}$	$I_6 = 3.33 \times 10^{-1} \pm 0.16 \times 10^{-1}$
$I_7 = 2.98 \times 10^{-1} \pm 0.29 \times 10^{-1}$	$I_8 = 2.14 \times 10^{-1} \pm 0.14 \times 10^{-1}$
$I_9 = 2.38 \times 10^{-1} \pm 1.29 \times 10^{-2}$	$I_{10} = -1.3 \times 10^{-2} \pm 0.22 \times 10^{-2}$
$I_{11} = 1.42 \times 10^{-2} \pm 0.70 \times 10^{-3}$	$I_{12} = 1.10 \times 10^{-2} \pm 0.11 \times 10^{-2}$
$I_{13} = -3.79 \times 10^{-2} \pm 0.90 \times 10^{-3}$	$I_{14} = 1.64 \times 10^{-2} \pm 0.07 \times 10^{-2}$
$I_{15} = 1.25 \times 10^{-1} \pm 0.30 \times 10^{-2}$	$I_{16} = 1.24 \times 10^{-1} \pm 0.10 \times 10^{-1}$
$I_{17} = 6.42 \times 10^{-2} \pm 3.00 \times 10^{-3}$	$I_{18} = 4.31 \times 10^{-2} \pm 3.30 \times 10^{-3}$
$I_{19} = 3.00 \times 10^{-2} \pm 14.0 \times 10^{-3}$	$I_{20} = 2.02 \times 10^{-2} \pm 1.80 \times 10^{-2}$
$I_{21} = -1.00 \times 10^{-2} \pm 6.00 \times 10^{-3}$	$I_{22} = -6.80 \times 10^{-3} \pm 4.50 \times 10^{-3}$
$I_{23} = 4.00 \times 10^{-2} \pm 2.00 \times 10^{-3}$	
$Im_1 = 1.14 \pm 0.27$	$Im_2 = 4.71 \pm 0.54$
$Im_3 = 5.27 \times 10^{-1} \pm 0.93 \times 10^{-1}$	$Im_4 = 2.00 \times 10^{-1} \pm 0.16 \times 10^{-1}$
$Im_5 = 1.70 \times 10^{-1} \pm 0.14 \times 10^{-1}$	$Im_6 = 1.35 \times 10^{-1} \pm 0.16 \times 10^{-1}$

(Gravitational constants have units of newton meters)

$g_1 = -37.2 \pm 00.5$	$g_2 = -8.44 \pm 0.20$
$g_3 = 1.02 \pm 0.50$	$g_4 = 2.49 \times 10^{-1} \pm 0.25 \times 10^{-1}$
$g_5 = -2.82 \times 10^{-2} \pm 0.56 \times 10^{-2}$	

Table A4. The expressions giving the elements of the kinetic energy matrix.
(The Abbreviated Expressions have units of kg-m².)

Table A5. The expressions giving the elements of the Coriolis matrix. (The Abbreviated Expressions have units of km^{-1} .)

Table A5. The expressions giving the elements of the Coriolis matrix. (The Abbreviated Expressions have units of $\text{kg}\cdot\text{m}^2$)

$$I_{12} = 2 * \{-I_3 * SC2 + I_3 * C223 + I_7 * SC23 - I_{12} * S223 + I_{13} * \{2 * SC23 * C5 + (1 - 2 * SS23) * C4 * S5\}$$

$$\begin{aligned}
& +I_{10} \cdot (C223 \cdot C5 - S223 \cdot C4 \cdot S5) + I_{11} \cdot SC23 \cdot C4 \cdot C4 \\
& + I_{12} \cdot ((-1 \cdot C4) \cdot SC23 \cdot S5 - (1 - 2 \cdot S232) \cdot C4 \cdot S5) \\
& + I_{13} \cdot ((1 - 2 \cdot S232) \cdot C5 - 2 \cdot SC23 \cdot C4 \cdot S5) \\
& + I_{10} \cdot (1 - 2 \cdot S232) + I_{11} \cdot (1 - 2 \cdot S232); \\
& \approx -2.76 \cdot S523 + 7.44 \cdot 10^{-4} \cdot C223 + 0.60 \cdot SC23 \\
& - 2.13 \cdot 10^{-3} \cdot (1 - 2 \cdot S232), \\
b_{115} = & 2 \cdot \{I_{10} \cdot C7 \cdot C23 + I_{11} \cdot SC23 - I_{12} \cdot C7 \cdot S23 \\
& + I_{13} \cdot (2 \cdot S23 \cdot C5 + (1 - 2 \cdot S232) \cdot C4 \cdot S5) \\
& + I_{14} \cdot C7 \cdot (C23 \cdot C5 - S23 \cdot S5) + I_{15} \cdot SC23 \cdot C4 \cdot C4 \\
& + I_{16} \cdot ((1 + C4) \cdot SC23 \cdot S5 - (1 - 2 \cdot S232) \cdot C4 \cdot S5) \\
& + I_{17} \cdot (1 - 2 \cdot S232) \cdot C5 - 2 \cdot S232 \cdot C4 \cdot S5\} \\
& + I_{10} \cdot (1 - 2 \cdot S232); \\
& \approx 7.44 \cdot 10^{-4} \cdot C7 \cdot C23 + 0.60 \cdot SC23 \\
& + 2.20 \cdot 10^{-3} \cdot C2 \cdot S23 - 2.13 \cdot 10^{-3} \cdot (1 - 2 \cdot S232), \\
b_{116} = & 2 \cdot \{-I_{13} \cdot SC23 \cdot S4 \cdot S5 - I_{16} \cdot C7 \cdot C23 \cdot S4 \cdot S5 \\
& - I_{18} \cdot C4 \cdot S5 - I_{19} \cdot (S232 \cdot S5 \cdot S4 - SC23 \cdot S4 \cdot SC5) \\
& + I_{12} \cdot (C23 \cdot S4 \cdot S5 - I_{11} \cdot S23 \cdot S4 \cdot C5) \\
& \approx -2.50 \cdot 10^{-3} \cdot SC23 \cdot S4 \cdot S5 + 8.60 \cdot 10^{-4} \cdot C4 \cdot S5 \\
& - 2.48 \cdot 10^{-3} \cdot C7 \cdot C23 \cdot S4 \cdot S5, \\
b_{117} = & 2 \cdot \{I_{10} \cdot (SC5 \cdot (C4 \cdot (1 - (C232) - (C232) \\
& - SC23 \cdot C4 \cdot (1 - 2 \cdot S232)) - I_{15} \cdot (S232 \cdot S5 - SC23 \cdot C4 \cdot C4 \\
& - I_{16} \cdot C7 \cdot (S23 \cdot S5 - C23 \cdot C4 \cdot S5) + I_{18} \cdot S4 \cdot C5 \\
& + I_{19} \cdot (C23 \cdot C4 \cdot C5 - SC23 \cdot S5))\} \\
& \approx -2.50 \cdot 10^{-3} \cdot (S232 \cdot S5 - SC23 \cdot C4 \cdot C5) \\
& - 2.48 \cdot 10^{-3} \cdot C7 \cdot (S23 \cdot S5 - C23 \cdot C4 \cdot C5) \\
& + 8.60 \cdot 10^{-4} \cdot S4 \cdot C5, \\
b_{118} = & 0, \\
b_{119} = & 2 \cdot \{-I_{13} \cdot S23 \cdot I_{10} \cdot C23 + I_{15} \cdot S23 \cdot S4 \cdot S5 \\
& - I_{16} \cdot (C23 \cdot C4 \cdot S5 + S23 \cdot C5) + I_{10} \cdot C23 \cdot SC4 \\
& + I_{17} \cdot S4 \cdot (C23 \cdot C4 \cdot C5 - S23 \cdot SC5) \\
& + I_{19} \cdot C23 \cdot S4 \cdot S5\}; \\
b_{120} = & -I_{13} \cdot S23 \cdot S4 \cdot S5 + I_{15} \cdot S23 \cdot (1 - (2 \cdot S234)) \\
& + I_{17} \cdot S23 \cdot (1 - 2 \cdot S234) + I_{19} \cdot S23 \cdot (1 - (2 \cdot S234)) \\
& + I_{10} \cdot S23 \cdot (1 - 2 \cdot S234) + I_{11} \cdot S23 \cdot (1 - (2 \cdot S234)) \\
& \approx 0, \\
b_{121} = & I_{17} \cdot C23 \cdot S4 \cdot I_{10} \cdot 2 \cdot (S23 \cdot C4 \cdot C5 - C23 \cdot S5) \\
& + I_{19} \cdot C23 \cdot (I_{15} \cdot S23 - S23 \cdot C4 \cdot 2 \cdot SC5) \\
& \approx 0, \\
b_{122} = & -I_{123} \cdot (S23 \cdot C5 + C23 \cdot C4 \cdot S5); \quad \approx 0, \\
b_{123} = & b_{124}, \quad b_{125} = b_{125}, \quad b_{126} = b_{126}, \\
b_{127} = & 2 \cdot \{I_{11} \cdot S23 \cdot C4 \cdot C5 + I_{10} \cdot C7 \cdot C4 \cdot S5 \\
& + I_{12} \cdot C23 \cdot S4 \cdot C5 + I_{13} \cdot C23 \cdot C4 \cdot C5\} + I_{17} \cdot S23 \cdot C4 \\
& - I_{19} \cdot (S23 \cdot C4 \cdot (1 - 2 \cdot S234) + 2 \cdot C23 \cdot SC5) \\
& \approx 0, \\
b_{128} = & I_{23} \cdot S23 \cdot S4 \cdot S5; \quad \approx 0, \\
b_{129} = & -I_{23} \cdot (C23 \cdot S5 + S23 \cdot C4 \cdot C5); \quad \approx 0, \\
b_{130} = & 0, \quad b_{131} = 0, \\
b_{132} = & I_{14} \cdot S23 + I_{15} \cdot S23 \cdot (1 - (2 \cdot S234)) \\
& + 2 \cdot (-I_{18} \cdot C23 \cdot C4 \cdot S5 + I_{19} \cdot S2 \cdot C4 \cdot S5) \\
& + I_{10} \cdot (S23 \cdot (C23 \cdot C4 \cdot C5 - 0.5) + S23 \cdot C4 \cdot SC5) \\
& - I_{12} \cdot S23 \cdot C4 \cdot S5\}; \\
& \approx 1.64 \cdot 10^{-4} \cdot S23 - 2.50 \cdot 10^{-3} \cdot C23 \cdot C4 \cdot S5 + \\
& 2.48 \cdot 10^{-3} \cdot S2 \cdot C4 \cdot S5 + 0.30 \cdot 10^{-3} \cdot S23 \cdot (1 - (2 \cdot S234)), \\
b_{133} = & 2 \cdot \{-I_{15} \cdot C23 \cdot S4 \cdot C5 + I_{10} \cdot S23 \cdot S4 \cdot C5 \\
& + I_{16} \cdot S2 \cdot S4 \cdot C5\} - I_{17} \cdot C23 \cdot S4 \\
& + I_{19} \cdot (C23 \cdot S4 \cdot (1 - 2 \cdot S232) - 2 \cdot S23 \cdot SC4 \cdot SC5) \\
& \approx -2.50 \cdot 10^{-3} \cdot C23 \cdot S4 \cdot C5 + 2.48 \cdot 10^{-3} \cdot S2 \cdot S4 \cdot C5 \\
& - 6.42 \cdot 10^{-4} \cdot S23 \cdot S4, \\
b_{134} = & -b_{126}, \\
b_{135} = & 2 \cdot \{-I_{123} \cdot S3 \cdot I_{11} \cdot C3 + I_{10} \cdot (C3 \cdot C5 - S3 \cdot C4 \cdot S5)\} \\
& \approx 2.20 \cdot 10^{-3} \cdot S3 + 7.44 \cdot 10^{-4} \cdot C3,
\end{aligned}$$

4 DOF robot arm

$b_{331} = b_{321} ,$	$b_{335} = b_{325} ,$	$b_{336} = 0 ,$
$b_{441} = b_{431} ,$	$b_{446} = 0 ,$	$b_{456} = 0 .$

Table A6. The expressions for the terms of the centrifugal matrix (The Abbreviated Expressions have units of kg-m².)

$$\begin{aligned}
c_{11} &= 0 , \\
c_{12} &= +I_4 \cdot C/2 - J_8 \cdot S/23 - J_9 \cdot S/2 + I_{13} \cdot C/23 \\
&\quad + I_{14} \cdot S/23 \cdot S_4 \cdot S_5 + I_{16} \cdot C/2 \cdot S_4 \cdot S_5 \\
&\quad + I_{18} \cdot (C/23 \cdot C_4 \cdot S_5 + S/23 \cdot C_5) + I_{19} \cdot C/23 \cdot S C_4 \\
&\quad + I_{20} \cdot S_4 \cdot (C/23 \cdot C_4 \cdot C/5 - S/23 \cdot S C_5) \\
&\quad + I_{21} \cdot S/23 \cdot S_4 \cdot S_5 ; \\
&\approx 6.90 \times 10^{-1} \cdot C/2 + 1.34 \times 10^{-1} \cdot S/23 - 2.38 \times 10^{-2} \cdot S/23 \\
&\approx 0.5 \cdot b_{123} , \\
c_{13} &= -I_{13} \cdot S/23 \cdot S_4 \cdot S_5 - I_{18} \cdot C/2 \cdot S_4 \cdot S_5 \\
&\quad + I_{19} \cdot (C/23 \cdot C_4 \cdot S_5 + S/23 \cdot S_4 \cdot S C_5) \\
&\quad - I_{22} \cdot C/23 \cdot S_4 \cdot S_5 ; \\
&\approx 0 , \\
c_{15} &= -I_{13} \cdot S/23 \cdot S_4 \cdot S_5 - I_{18} \cdot C/2 \cdot S_4 \cdot S_5 ; \\
&\quad + I_{18} \cdot (C/23 \cdot C_5 \cdot C/23 \cdot C_4 \cdot S_5) - I_{22} \cdot C/23 \cdot S_4 \cdot S_5 \\
&\approx 0 , \\
c_{16} &= 0 , \quad c_{21} = -0.5 \cdot b_{112} , \\
c_{22} &= 0 , \quad c_{23} = 0.5 \cdot b_{223} , \\
c_{24} &= -I_{13} \cdot C/4 \cdot S_5 - I_{18} \cdot S/23 \cdot C_4 \cdot S_5 + I_{20} \cdot C/4 \cdot S C_5 ; \\
&\approx 0 , \\
c_{25} &= -I_{13} \cdot C/4 \cdot S_5 + I_{16} \cdot (C/3 \cdot C_5 - S/3 \cdot C/4 \cdot S_5) \\
&\quad + I_{22} \cdot C/5 ; \quad \approx 0 , \\
c_{26} &= 0 , \quad c_{31} \approx -0.5 \cdot b_{113} , \\
c_{32} &= -c_{23} , \quad c_{33} = 0 , \\
c_{34} &= -I_{13} \cdot C/4 \cdot S_5 + I_{20} \cdot C/4 \cdot S C_5 ; \\
&\approx -1.26 \times 10^{-3} \cdot C/4 \cdot S_5 , \\
c_{35} &= -I_{13} \cdot C/4 \cdot S_5 + I_{22} \cdot C/5 ; \quad \approx c_{34} , \\
c_{36} &= 0 , \quad c_{41} = -0.5 \cdot b_{116} , \quad c_{42} = -0.5 \cdot b_{117} , \\
c_{43} &= 0.5 \cdot b_{223} , \quad c_{44} = 0 , \quad c_{45} = 0 , \\
c_{46} &= 0 , \quad c_{51} = -0.5 \cdot b_{115} , \quad c_{52} = -0.5 \cdot b_{116} , \\
c_{53} &= 0.5 \cdot b_{222} , \quad c_{54} = -0.5 \cdot b_{445} , \quad c_{55} = 0 , \\
c_{56} &= 0 , \quad c_{61} = 0 , \quad c_{62} = 0 , \\
c_{63} &= 0 , \quad c_{64} = 0 , \quad c_{65} = 0 , \\
c_{66} &= 0 .
\end{aligned}$$

Table A7. Gravity Terms.
(The Abbreviated Expressions have units of newton-meters.)

$$\begin{aligned}
E_1 &= 0 , \\
E_2 &= g_1 \cdot C/2 + g_2 \cdot S/23 + g_3 \cdot S/2 + g_4 \cdot C/23 ; \\
&\quad + g_5 \cdot (S/23 \cdot C/5 + C/23 \cdot C/4 \cdot S_5) \\
&\approx -37.2 \cdot C/2 - 8.4 \cdot S/23 + 1.02 \cdot S/2 , \\
E_3 &= g_2 \cdot S/23 + g_4 \cdot C/23 + g_5 \cdot (S/23 \cdot C/5 + C/23 \cdot C/4 \cdot S_5) ; \\
&\approx 8.4 \cdot S/23 + 0.26 \cdot C/23 , \\
E_4 &= -g_5 \cdot S/23 \cdot S_4 \cdot S_5 ; \\
&\approx 2.8 \times 10^{-1} \cdot S/23 \cdot S_4 \cdot S_5 , \\
E_5 &= g_5 \cdot (C/23 \cdot S_5 + S/23 \cdot C/4 \cdot C_5) ; \\
&\approx -2.8 \times 10^{-3} \cdot (C/23 \cdot S_5 + S/23 \cdot C/4 \cdot C_5) , \\
E_6 &= 0 ,
\end{aligned}$$

Table A7. Gravity Terms.
[The Abbreviated Expressions have units of newton-meter]

$$\begin{aligned} R_1 &= 0, \\ R_2 &= g_1 \cdot C2 + g_2 \cdot S23 + g_3 \cdot S2 + g_4 \cdot C23; \\ &+ g_5 \cdot (S23 \cdot C5 + C23 \cdot C4 \cdot S5) \\ &\approx -37.2 \cdot C2 - 8.4 \cdot S23 + 1.02 \cdot S2, \\ R_3 &= g_2 \cdot S23 + g_4 \cdot C23 + g_5 \cdot (S23 \cdot C5 + C23 \cdot C4 \cdot S5); \\ &\approx -8.4 \cdot S23 + 0.25 \cdot C23, \\ R_4 &= -g_5 \cdot S23 \cdot S4 \cdot S5; \\ &\approx 2.8 \times 10^{-3} \cdot S23 \cdot S4 \cdot S5, \\ R_5 &= g_5 \cdot (C23 \cdot S5 + S23 \cdot C4 \cdot C5); \\ &\approx -2.8 \times 10^{-3} \cdot (C23 \cdot S5 + S23 \cdot C4 \cdot C5), \\ R_6 &= 0. \end{aligned}$$

Controller Formulation - ISM

Three Layer Control Architecture

1) Inner Loop: Inverse Dynamics Controller

$$\ddot{\mathbf{q}} = \mathbf{v} - \boldsymbol{\eta}(\mathbf{q}, \dot{\mathbf{q}}) \quad \boldsymbol{\eta}(\mathbf{q}, \dot{\mathbf{q}}) = -\mathbf{M}^{-1}(\mathbf{q})(\hat{\mathbf{n}}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}})) .$$

N-decoupled

Double Integrators

$$\begin{cases} \dot{x}_{1_i}(t) = x_{2_i}(t) \\ \dot{x}_{2_i}(t) = v_i(t) - \eta_i(t) \end{cases}$$

2) Middle Loop: Integral Sliding Mode (ISM)

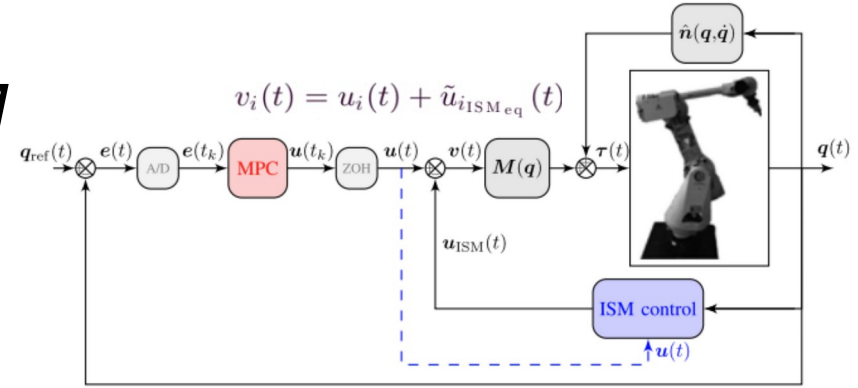
$$\tilde{u}_{i\text{ISMeq}}(t) = \frac{1}{\mu_i} \int_{t_0}^t e^{-\frac{1}{\mu_i}(t-\zeta)} u_{i\text{ISM}}(\zeta) d\zeta$$

$$\begin{cases} \dot{x}_{1_i}(t) = x_{2_i}(t) \\ \dot{x}_{2_i}(t) = u_i(t) \end{cases}$$

$$\sigma_i(\mathbf{x}_i(t)) =$$

$$\mathbf{S}_i \left(\mathbf{x}_i(t) - \mathbf{x}_i(t_0) - \int_{t_0}^t [x_{2_i}(\zeta), v_i(\zeta) - u_{i\text{ISM}}(\zeta)]^T d\zeta \right)$$

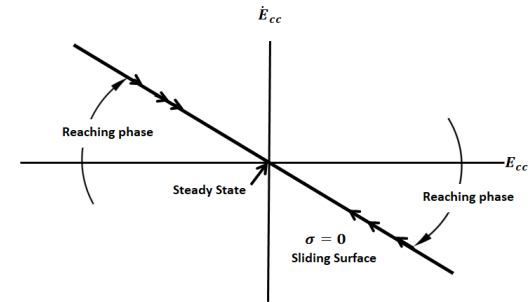
$$\dot{\mathbf{x}}_i(t) = \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{B}_i u_i(t)$$



Sliding Mode Control

$$\sigma_i = \mathbf{S}_i \left[\mathbf{x}_i(t) - \mathbf{x}_i(t_0) - \int_{t_0}^t \begin{bmatrix} x_{2_i}(\zeta) \\ u_i(\zeta) \end{bmatrix} d\zeta \right], \quad \mathbf{S}_i = [c_i \ 1]$$

$$u_i^{\text{ISM}} = -U_i^{\max} \text{sgn}(\sigma_i)$$



Sliding surface and sliding variable. [2]

Controller Formulation - MPC

Outer Loop: MPC

Recan: $\ddot{\mathbf{q}} = \mathbf{v} + \mathbf{M}^{-1}(\mathbf{q})(\hat{\mathbf{n}}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}})) = \mathbf{v} - \boldsymbol{\eta}(\mathbf{q}, \dot{\mathbf{q}})$

$$\mathbf{v}_i(t) = \mathbf{u}_i(t) + \tilde{\mathbf{u}}_{i\text{ISM}_{\text{eq}}}(t)$$

$$\mathbf{x}_{k+1} = \tilde{\mathbf{A}} \mathbf{x}_k + \tilde{\mathbf{B}} \mathbf{u}_k, \quad \tilde{\mathbf{A}} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \tilde{\mathbf{B}} = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$$

Cost Function:

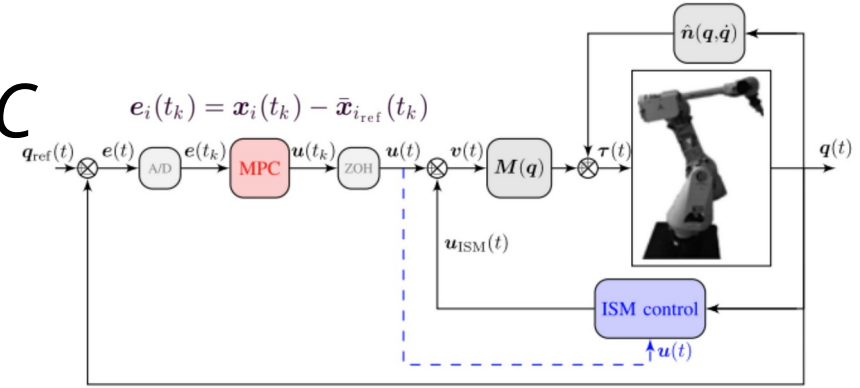
$$J(\mathbf{e}_i(t_k), \mathbf{u}_{i[t_k, t_k+N-1|t_k]}, N) =$$

$$\sum_{j=0}^{N-1} \|\mathbf{e}_i(t_{k+j})\|_{\mathbf{Q}_i}^2 + \|\mathbf{u}_i(t_{k+j})\|_{\mathbf{R}_i}^2 + \|\mathbf{e}_i(t_{k+N})\|_{\mathbf{\Pi}_i}^2$$

Subject to: $\mathbf{x}_i(t_{k+j}) \in \mathcal{X}$

$$\mathbf{x}_i(t_{k+N}) \in \mathcal{X}_f$$

$$\|\mathbf{u}_i(t_{k+j})\| \leq v_{i\text{max}} - U_{i\text{max}}$$



Terminal Set and Terminal Control:

$$\mathcal{X}_f := \{\mathbf{x}_i \mid \|\mathbf{x} - \bar{\mathbf{x}}_{i\text{ref}}\|_{\mathbf{\Pi}}^2 \leq \rho\}, \quad \mathcal{X}_f \subseteq \mathcal{X}$$

K_{LQ} is the control gain of an infinite horizon Linear Quadratic (LQ) controller with the same cost function

$$\kappa_{i_f}(\mathbf{e}_i(t_k)) = \mathbf{K}_{LQ} \mathbf{e}_i(t_k)$$

$$\mathbf{x}_i(t_k) \in \mathcal{X}_f$$

$$\|\kappa_{i_f}(\mathbf{e}_i(t_k))\| \leq v_{i\text{max}} - U_{i\text{max}}$$

Solution to the Riccati Equation

$$(\tilde{\mathbf{A}}_i - \tilde{\mathbf{B}}_i \mathbf{K}_{LQ})^T \mathbf{\Pi}_i (\tilde{\mathbf{A}}_i - \tilde{\mathbf{B}}_i \mathbf{K}_{LQ}) - \mathbf{\Pi}_i = -\mathbf{Q}_i - \mathbf{K}_{LQ}^T \mathbf{R}_i \mathbf{K}_{LQ}$$

Paper Results

- 3-joint COMAU Smart3-S2 industrial robot manipulator
- Realistic uncertainty injection (identified from real robot data)
- Constraints: See Table I (position, velocity, acceleration limits)
- Target position: $q_{ref} = [\pi/4, \pi/3, 2\pi/4]^T$, from $q_0 = [0, 0, 0]$
- Controller parameters:
 - $C_i = [10, 10, 10]$
 - ISM gains: $[20, 35, 85]$
 - MPC: $Q = \text{diag}(100, 100)$, $R = 0.1$,
 - Terminal Weight
 - Horizon $N=10$
 - Sampling: MPC = 20ms, ISM = 1ms

Key Results:

- MPC/ISM reduces RMS error dramatically:
 - Joint 1: $0.2105 \rightarrow 0.0070$ rad (30× decrease)
 - Joint 3: $1.0458 \rightarrow 0.0152$ rad (69× decrease)
- Similar control effort but better performance
- Computational efficiency:
 - MPC: 18ms average execution time
 - ISM: only 29μs average execution time
 - Suitable for real-time implementation
- MPC alone violates constraints,
- MPC/ISM ensures constraint satisfaction

TABLE I
STATE AND INPUT CONSTRAINTS FOR EACH JOINT

Joint i	$q_{i\max}$ (rad)	$\dot{q}_{i\max}$ (rad s ⁻¹)	$v_{i\max}$ (rad/s ²)
1	1.83	2	145
2	2.71	3.5	250
3	3.49	6.3	350

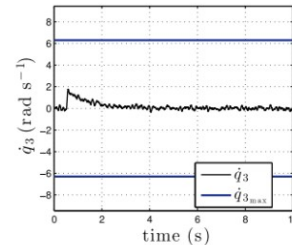
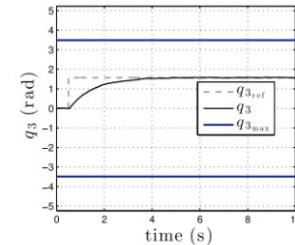
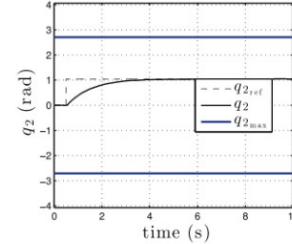
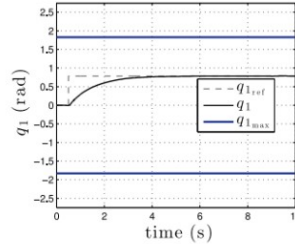
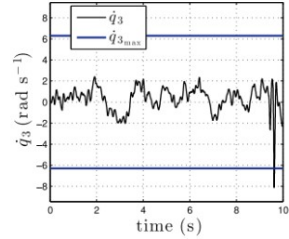
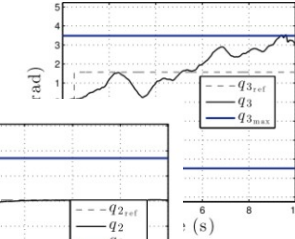
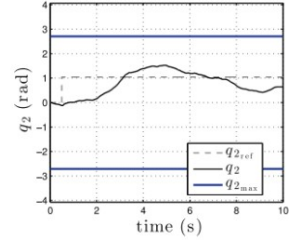
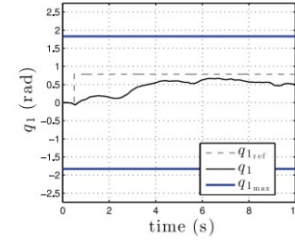


TABLE II
PERFORMANCE INDEXES FOR EACH JOINT

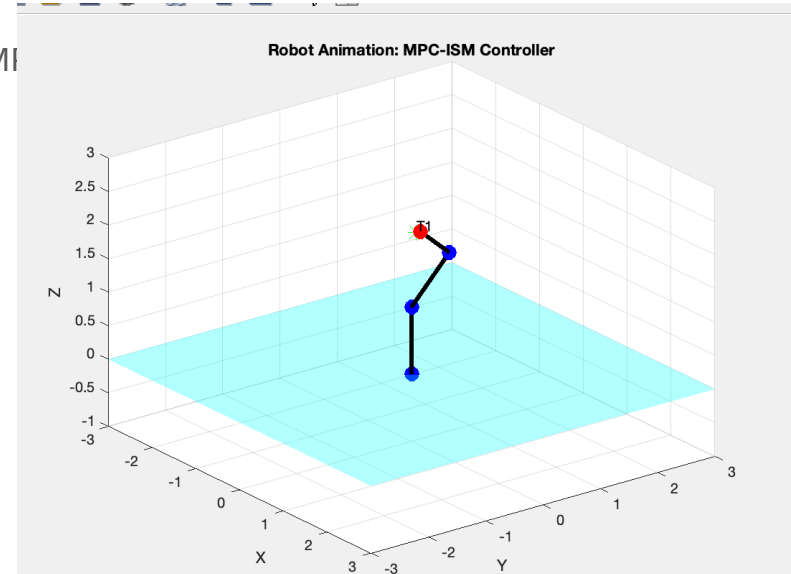
Strategy	Joint i	e_{RMS_i} (rad)	E_{c_i} (rad/s ²)
MPC	1	0.2105	8.0303
	2	0.3749	13.4839
	3	1.0458	41.2172
MPC/ISM	1	0.0070	8.0600
	2	0.0102	13.4217
	3	0.0152	34.8530

TABLE III
TIME CONSUMPTION OF THE PROPOSED CONTROL STRATEGY IN SECONDS

Algo.	Mean	Min.	Max.	Std. dev.
MPC	0.018	0.017	0.71	0.011
ISM	2.9×10^{-5}	2.7×10^{-5}	0.0086	1.4×10^{-4}

Implementation on Matlab

- 1) Robot Joint Angle target : $\pi/2$, $\pi/3$, $\pi/4$
- 2) Joint trajectory from MPC and MPC-ISM controllers compared to reference cubic polynomial trajectory
- 3) Sinusoidal noise injected to see the benefits of the MPC-ISM Combined Controller Algorithm



Uncertainty Formulation and Injection

1. Uncertainty formulation:

```
% Deterministic component based on position
pos_comp = 0.5 * max_eta * sin(q(i));

% Deterministic component based on velocity
vel_comp = 0.2 * max_eta * sign(q_dot(i)) * min(abs(q_dot(i)), 1);
|
% Random component (bounded)
rand_comp = 0.3 * max_eta * (2*rand() - 1);

% Combined uncertainty
eta(i) = pos_comp + vel_comp + rand_comp;
```

2. Uncertainty injection:

Where M is the inertia matrix,
 n contains the nonlinear terms
(friction, coriolis effect), and η is
the uncertainty

$$\ddot{q} = M^{-1}(u - n + \eta)$$

```
% Compute acceleration: q_ddot = M^-1(\tau - n + \eta)|
q_ddot = M \ (u - n + eta);
```

3. This is done for all 3 joints.

Results

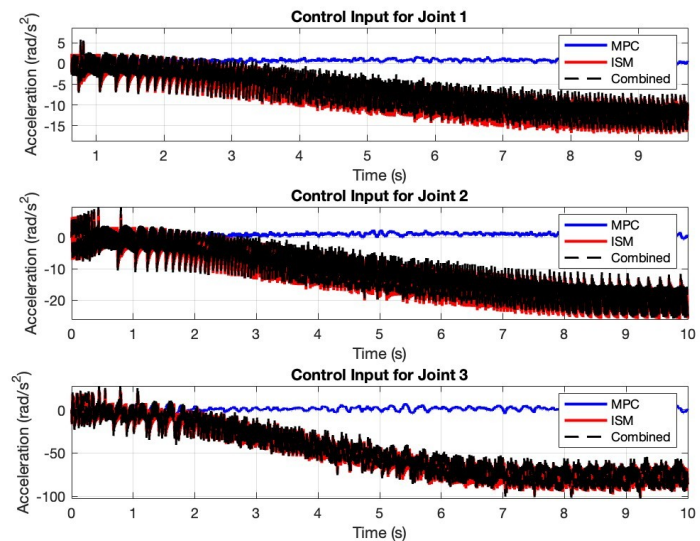
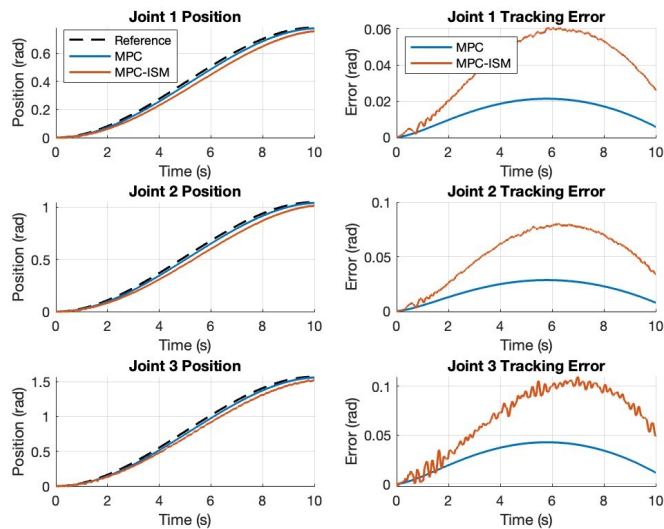
- 1) The results were not what was expected based on the paper.
- 2) MPC Controller was very well tuned to account for the noise and gave highly accurate results.
- 3) MPC-ISM performed slightly worse than MPC.
- 4) However, the difference seems negligible - both controllers have an error within hundredths of a radian.

```
Controller: MPC
RMS Tracking Error (rad):
  Joint 1: 0.0156
  Joint 2: 0.0207
  Joint 3: 0.0311
Control Effort (rad/s^2):
  Joint 1: 0.0259
  Joint 2: 0.0345
  Joint 3: 0.0518
Steady-state Error (rad):
  Joint 1: 0.005942
  Joint 2: 0.007923
  Joint 3: 0.011884
```

```
Controller: MPC-ISM
RMS Tracking Error (rad):
  Joint 1: 0.0434
  Joint 2: 0.0571
  Joint 3: 0.0748
Control Effort (rad/s^2):
  Joint 1: 8.4462
  Joint 2: 13.5679
  Joint 3: 57.2287
Steady-state Error (rad):
  Joint 1: 0.026736
  Joint 2: 0.034350
  Joint 3: 0.050732
```

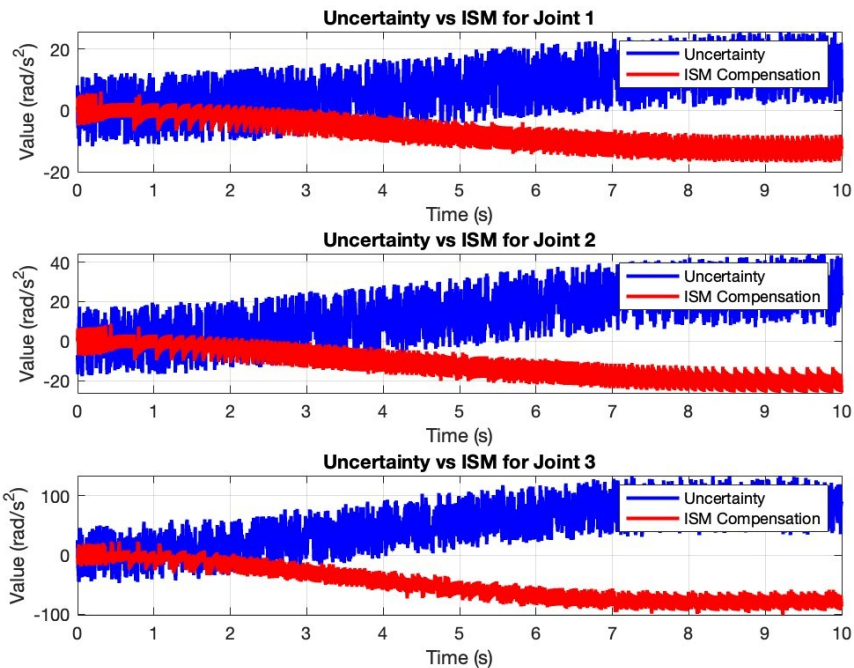
Results

- 1) We can see however, that the ISM Controller adjusts the control input for the joints to account for the uncertainty



Results

- 1) As the uncertainty goes higher in magnitude, the ISM controllers counter action increases as well



Possible considerations

- 1) Uncertainty modeling - The simulated uncertainty might have not been aggressive enough to impact MPC performance as the uncertainty injected by the authors
- 2) Hardware Differences: The COMAU Smart3-S2 robot in the paper may have different characteristics than our simulated model
 - a) The paper notes that their model of the COMAU Smart3-S2 on Matlab Simulink was based on data from real experiments.
 - b) However, we are simulating a perfect model on which the MPC acts upon. Due to this, the performance of MPC itself is highly accurate.
 - c) The performance difference between MPC and ISM is marginally different, and seems to be negligible due to this .

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- [2] Adaptive chaos control of a humanoid robot arm: a fault-tolerant scheme - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Sliding-surface-and-sliding-variable_fig1_370320084
- [3] *ECE 5463* taught at Ohio State University by Prof. Wei Zhang https://storage1.ucsd.edu/slides/CSE291Robo/L9_lagrangian.html#/title